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Epistemological Rupture in the Transition from Frequency to Formalism: An Analysis of the Probability Concept in Chinese Mathematics Curriculum

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Abstract

The probability concept occupies an increasingly central position in school mathematics, yet its development across educational levels involves discontinuities that resist smooth cognitive progression. In the Chinese mathematics curriculum, the transition from junior to senior high school requires students to move from a "frequency intuition" conception of probability, where probability is understood as the long-run relative frequency of events observed through repeated experiments, to a "random variable" conception, where probability is formalized through sample spaces, events as sets, and random variables as numerical functions defined on outcomes. This paper argues that such a transition constitutes what Bachelard (1938) termed an epistemological rupture, a break in which previously adequate ways of knowing become obstacles to new knowledge. Through a theoretical analysis grounded in Bachelard's epistemological obstacle, Duval's theory of semiotic representation registers, and Sfard's reification framework, this study identifies three interrelated dimensions of the rupture: ontological (the shift from probability as an empirical property of experiments to probability as a formal measure on abstract mathematical structures), semiotic (the shift from verbal and numerical descriptions of experiments to the formal language of set theory and random variables), and cognitive (the demand for a structural conception of probability built on inadequate empirical foundations). By examining textbook definitions, curriculum standards, and the mathematical structure of the concept itself, this paper traces the specific mechanisms through which prior knowledge becomes an impediment and proposes that recognizing this rupture as structurally necessary can inform more coherent instructional design at the critical juncture between school levels.

I. INTRODUCTION

Probability is among the mathematical ideas that students encounter in seemingly familiar forms yet must ultimately reconstruct at a far more abstract level. From the everyday intuition that some events are "more likely" than others, to the systematic apparatus of sample spaces, random variables, and distributions, the probability concept serves as both a bridge between mathematics and lived experience and a persistent source of cognitive difficulty. Students encounter probability repeatedly across grade levels, yet each encounter demands not merely an extension but, in certain respects, a fundamental restructuring of what probability is understood to be.

In China's mathematics curriculum, this restructuring takes a particularly sharp form. During the junior high school years (grades 7 to 9), students develop a conception of probability centered on the idea of *frequency* and *classical probability*. The 2024 edition of the junior high mathematics textbook published by the People's Education Press (人教版 2024, hereafter PEP-JH) presents probability through concrete experiments, expressing the idea that when an experiment is performed many times, the frequency of an event gradually stabilizes around a fixed value, which is taken as the probability of that event (People's Education Press, 2024). Students at this level compute probabilities using the classical formula $P(A) = m/n$, where m is the number of favorable outcomes and n is the total number of equally likely outcomes. They work with coin tosses, dice rolls, card draws, and spinning wheels, always anchored in concrete physical experiments whose outcomes can be observed and counted.

When students enter senior high school (grade 10 onward), they encounter a fundamentally different framework. The senior high textbook (人教 A 版 2019, hereafter PEP-SH) introduces set-theoretic language, defining the result of a random experiment as a *sample point* ω, the set of all sample points as the *sample space* Ω, and a *random event* as a subset of the sample space (People's Education Press, 2019a). Probability is now formalized through the language of sets: the sample space Ω, events as subsets of Ω, and probability as a function P assigning a number between 0 and 1 to each event, satisfying the axioms $P(\Omega) = 1$ and additivity for mutually exclusive events. Moreover, the concept of *random variable* is introduced: a numerical function $X: \Omega \to \mathbb{R}$ that assigns a real number to each sample point (People's Education Press, 2019b). Students are now expected to work with discrete random variables, compute their probability distributions, expected values, and variances, and apply the binomial and normal distributions.

The gap between these two formulations is not merely a matter of increasing precision. The junior high conception treats probability as an empirical property of experiments: the probability of getting heads is 0.5 because in many tosses, heads appear about half the time. The senior high conception treats probability as a formal measure on a set-theoretic structure, where events are subsets, random variables are functions, and probability is a mapping that satisfies abstract axioms. A student who has spent three years understanding probability through experiments and frequency must now understand it through sets and formal structures.

This shift has been recognized in Chinese mathematics education research as a practical difficulty (e.g., Qu, 2016; He, 2020), yet it has rarely been subjected to systematic theoretical analysis using epistemological frameworks. Most discussions treat it as a pedagogical challenge to be managed through more experiments or more practice with formulas. Such an approach, however well-intentioned, may underestimate the depth of the cognitive reorganization required.

This paper takes a different position. Drawing on Bachelard's (1938) concept of epistemological rupture, we argue that the transition from frequency intuition to random variable formalism is not a continuous deepening of understanding but a genuine rupture: a break in which the very knowledge that served the student well in junior high school becomes an obstacle to learning at the senior high level. To analyze this rupture, we employ three complementary theoretical lenses: Bachelard's theory of epistemological obstacles (as developed in mathematics education by Brousseau, 1983, 1997), Duval's (1995, 2006) theory of semiotic representation registers, and Sfard's (1991, 2000) theory of reification. Together, these frameworks allow us to examine the rupture at multiple levels: the epistemological structure of the obstacle, the semiotic shift in representational systems, and the cognitive transformation from process to object.

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The analysis proceeds as follows. Section 2 presents the three theoretical frameworks and explains how they converge to illuminate concept transitions. Section 3 describes the specific curricular context of probability teaching in China, drawing on the most recent textbook editions and curriculum standards. Section 4 constitutes the core analysis, examining the epistemological rupture through three dimensions. Section 5 discusses the implications for teaching and curriculum design, and Section 6 offers concluding remarks.

II. THEORETICAL FRAMEWORK

2.1 Knowledge as Its Own Barrier: Bachelard's Framework

The central claim of this paper, that prior knowledge can impede rather than support further learning, draws on a tradition in the philosophy of science that treats cognitive difficulty as endogenous to knowledge rather than exogenous to it. Gaston Bachelard (1938/2002) captured this idea in his notion of the *epistemological obstacle* (*obstacle épistémologique*). Bachelard's provocation was simple but far-reaching: the mind does not approach new knowledge as a blank slate, nor are the main barriers to understanding always external (complex problems, inadequate instruments, poor teaching). Often, the barrier comes from within knowledge itself. A way of thinking that served well in one context becomes a constraint when the learner encounters a situation that demands a fundamentally different mode of thought.

For the probability concept, Bachelard's framework is especially apt. Probability is not a concept that students encounter for the first time at school; they arrive with everyday intuitions about chance, luck, and likelihood. The curriculum then provides an initial formalization (probability as experimental frequency), which works well for a range of problems. But this very success creates a localized, context-bound conception that can resist generalization. Bachelard's typology of obstacles helps us specify what goes wrong: the *obstacle of first experience* (the tendency to generalize from the initial kind of encounter (in this case, repeated experiments with physical objects)); the *verbal obstacle* (the confusion generated when everyday terms like "event" or "random" acquire precise technical meanings that partially overlap with, but fundamentally differ from, their ordinary usage); and the *substantialist obstacle* (the difficulty of treating a process (observing frequency in experiments) as an object (a measure defined on an abstract structure)).

Guy Brousseau (1983, 1997) transposed Bachelard's epistemology into a didactic framework, arguing that obstacles in mathematics learning are not artifacts of poor instruction but are inherent in the structure of mathematical knowledge itself. The teacher's task, in Brousseau's view, is not to prevent students from encountering obstacles but to orchestrate conditions under which obstacles can be recognized and overcome, a process Bachelard called the *rectification* of knowledge (*connaître, c'est rectifier une connaissance*).

The related concept of *epistemological rupture* names the positive side of this dynamic. If the obstacle is the resistance encountered when old knowledge proves inadequate, the rupture is the break that must occur for understanding to advance. The rupture is not a failure but a restructuring: prior knowledge is neither discarded nor smoothly extended but fundamentally reorganized. Bachelard's insistence that all knowledge amounts to a rectification of prior knowledge (Bachelard, 1938/2002) means that intellectual progress requires a productive negation of earlier modes of thought, modes that were themselves necessary at an earlier stage of development.

2.2 The Semiotic Dimension: How Representations Shape and Constrain Understanding

A concept transition is never purely conceptual; it is always also a shift in the systems of signs through which the concept is expressed and manipulated. Duval's (1995, 2006) theory of semiotic representation registers provides the tools to analyze this dimension. Duval's starting point is the recognition that mathematical objects have no direct perceptual existence: they can only be accessed through their representations. A probability is not seen or touched; what is seen and touched are frequency tables, probability histograms, set-theoretic expressions, and distribution graphs. The relationship between the concept and its representations is not incidental but constitutive.

For the present analysis, two of Duval's distinctions are especially important. The first is between *treatment* (transforming a representation within the same register, for instance, simplifying a probability expression) and *conversion* (moving between registers, for instance, translating a verbal description of an experiment into set-theoretic notation). Conversion is more cognitively demanding than treatment and is, in Duval's view, the critical test of genuine understanding. The second is *congruence*: two representations in different registers are congruent when their structural elements correspond one-to-one. High congruence makes conversion relatively transparent; low congruence makes it opaque and effortful.

When a curriculum transition introduces new registers (from verbal descriptions to set-theoretic notation, from frequency tables to distribution functions) and the congruence between old and new registers is low, the resulting difficulty is not merely a matter of learning new notation; it is a genuine *rupture in semiosis*, a disconnection between the representational activity the student can perform and the conceptual activity the new situation demands.

2.3 From Doing to Having: The Reification of Mathematical Concepts

Many mathematical concepts exhibit what Sfard (1991, 2000) called a *dual nature*: they can be apprehended either as processes (something the learner *does*) or as objects (something the learner *has*). The probability concept illustrates this duality with particular clarity. At one level, "doing probability" means performing experiments, observing outcomes, tallying frequencies, and computing numerical values. At another level, probability is a static mathematical entity, a measure on a sample space that can be manipulated, compared, and subjected to formal operations. The transition between these two modes is what Sfard termed *reification*: the transformation of a process into a conceptual object.

Sfard described this transformation in three stages. *Interiorization* is the familiarization with a concept at the operational level: the student learns to carry out the relevant processes competently (conducting experiments, counting outcomes, applying formulas). *Condensation* is the ability to perceive the process as a whole without executing each step; the student can reason about probability without performing the experiment, imagining what would happen in principle rather than actually doing it. *Reification* is the decisive breakthrough: the process is suddenly apprehended as a manipulable object, an ontological shift in which probability becomes something to reason *about* rather than something to reason *through*.

Reification is not gradual. Sfard (1991) described it as a sudden intellectual restructuring in which the concept emerges when the process is apprehended as an object. The difficulty lies in the fact that the operational conception, far from being a stepping stone to the structural conception, becomes the very thing that blocks it. The student who has invested years in building a robust process conception of probability, anchored in experiments, grounded in frequency, and tied to concrete physical situations, must undergo an *ontological shift* in which all of that operational competence is reorganized into a new kind of understanding. This shift is precisely where Bachelard's obstacle and Sfard's reification converge: the operational conception that enabled the student's earlier progress now functions as an impediment to further development.

2.4 Toward an Integrated Analytical Lens

Each of the three frameworks captures a dimension of concept transition that the others leave partially visible. Bachelard identifies the *epistemological structure* of the difficulty: why knowledge that was once adequate becomes an impediment. Duval identifies the *representational mechanism*: how shifts in sign systems both enable and constrain understanding. Sfard identifies the *cognitive transformation*: the shift from operational to structural conception that the transition demands. No single framework captures all three dimensions simultaneously, and no single

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framework can account for the specific difficulties that arise in the probability concept transition.

Used together, however, they provide mutually reinforcing explanatory power. The epistemological obstacle explains *why* the frequency intuition conception resists restructuring; the semiotic analysis explains *how* the shift in representational registers amplifies the difficulty; and the reification framework explains *what* kind of cognitive transformation is required. For the probability concept specifically, this convergence reveals a distinctive challenge: the student must not only restructure what probability means (ontological rupture), but also learn to work in entirely new representational systems (semiotic rupture), and simultaneously transform a process of experimental observation into a formal mathematical object (cognitive rupture). These three ruptures are not independent; they interact, reinforce each other, and together constitute the full epistemological break that the curriculum transition demands.

III. THE PROBABILITY CONCEPT IN THE CHINESE MATHEMATICS CURRICULUM

3.1 Junior High School: The Frequency Intuition Conception

In the Chinese mathematics curriculum, the probability concept is first formally introduced in grade 9 (junior high year 3), within the "Statistics and Probability" domain. The 2022 edition of the Compulsory Education Mathematics Curriculum Standards specifies that students should develop an understanding of probability through experiments, learn to compute probabilities of simple events using the classical formula, and recognize how frequency relates to probability (Ministry of Education, 2022).

The PEP-JH textbook (2024 edition, Volume 2, Chapter 25) presents the probability concept through a sequence of concrete experiments. The first experiments involve coin tossing: students are asked to toss a fair coin 50 times, record the number of heads, and compute the ratio of heads to total tosses, which the textbook terms the *frequency* of the event "heads" (People's Education Press, 2024). Students then perform similar experiments with dice, colored balls drawn from bags, and spinning wheels, computing frequencies for various events.

After presenting several such experiments, the textbook abstracts a common pattern and introduces the classical probability formula: for an experiment with n equally likely outcomes, the probability of an event A consisting of m outcomes is $P(A) = m/n$ (People's Education Press, 2024).

This definition emphasizes several features. First, probability is understood through *experiment*: it is grounded in the outcomes of repeated physical trials. Second, the emphasis is on *frequency*: probability is the long-run stable value that frequency approaches. Third, the classical formula provides a computational shortcut when outcomes are equally likely. The student's understanding at this level can be characterized as *frequency intuition*: probability is a number that describes how often an event occurs in repeated experiments, computable by counting favorable outcomes.

During grade 9, students study several types of probability problems: classical probability (coin tosses, dice rolls, card draws), geometric probability (areas and lengths), and simple compound events using tree diagrams and tables (Ministry of Education, 2022; People's Education Press, 2024). Throughout, the emphasis remains on the empirical connection between probability and observable experiments: students conduct experiments, record frequencies, compare them with theoretical probabilities, and observe the "law of large numbers" informally.

The cognitive achievement at the end of junior high school is substantial. Students can compute classical probabilities, construct sample spaces for simple experiments using lists or tree diagrams, estimate probabilities from experimental data, and solve problems involving compound events. Yet this achievement rests entirely on the frequency intuition conception. Probability is a property of experiments, computed by counting outcomes or observing frequencies, expressed through natural language and

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simple arithmetic, and understood primarily through its connection to physical randomness.

3.2 Senior High School: The Random Variable Conception

When students enter senior high school (grade 10 and beyond, particularly in the elective module "Probability and Statistics"), the probability concept undergoes a fundamental redefinition. The PEP-SH textbook (2019 edition, Compulsory Volume 2 and Selective Compulsory Volumes 2–3) revisits the junior high conception before moving to a more formal framework:

The textbook acknowledges the junior high approach, studying probability through experiments and frequency, before introducing set-theoretic language as a more rigorous foundation (People's Education Press, 2019a). The new framework defines the result of a random experiment as a *sample point* ω, the set of all sample points as the *sample space* Ω, and a *random event* as a subset of Ω. The probability of an event A is defined as a real number $P(A)$ satisfying three conditions: non-negativity ($P(A) \geq 0$), normalization ($P(\Omega) = 1$), and additivity for mutually exclusive events ($P(A \cup B) = P(A) + P(B)$) (People's Education Press, 2019a).

This formulation differs from the junior high framework in several fundamental respects. First, the ontological status of probability has changed. Instead of a property of experiments derived from frequency, probability is now a *measure* defined on a set-theoretic structure. The sample space Ω, events as subsets of Ω, and probability as a function satisfying the axioms; these are abstract mathematical objects, not descriptions of physical experiments.

Second, the language has shifted from the informal register of experiments and frequency to the formal register of set theory. Terms such as "sample space" (样本空间), "sample point" (样本点), "event" (事件 as subset), "mutually exclusive" (互斥), and "independent" (独立) replace the language of "tossing coins," "drawing balls," and "frequency stabilizing."

Third, and most importantly for the rupture analysis, the concept of *random variable* is introduced. The textbook defines a random variable X as a function that assigns a real number to each sample point in the sample space, that is, $X: \Omega \to \mathbb{R}$ (People's Education Press, 2019b).

Students are then expected to work with discrete random variables, their probability distributions (listing all possible values and their probabilities), expected values $E(X) = \Sigma x_i p_i$, variances $D(X) = \Sigma(x_i - E(X))^2 p_i$, and special distributions (binomial, hypergeometric, normal). The emphasis shifts from *observing frequencies in experiments* to *analyzing mathematical properties of random variables*.

3.3 The Curricular Gap

The two curriculum documents that govern probability teaching at each level make the discontinuity explicit. The Compulsory Education Mathematics Curriculum Standards (2022 edition) requires students to develop an understanding of probability through experiments and to recognize how frequency relates to probability (Ministry of Education, 2022), emphasizing intuitive understanding and empirical grounding. The Senior High School Mathematics Curriculum Standards (2017 edition, revised 2020) requires students to understand the notions of sample space and events, apply the axioms of probability, work with random variables and their distributions, and compute expected values and variances (Ministry of Education, 2020), emphasizing formal definition and abstract reasoning.

Between these two documents lies a significant conceptual gap: the junior high curriculum cultivates an empirical, experimentally grounded understanding, while the senior high curriculum demands a formal, set-theoretically grounded understanding. The gap is not a matter of degree (more depth, more complexity) but of kind: the very conception of what probability *is* must be restructured.

IV. ANALYSIS OF THE EPISTEMOLOGICAL RUPTURE

4.1 The Ontological Rupture: From Empirical Property to Formal Measure

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The most fundamental dimension of the rupture is ontological. At the junior high level, probability is an *empirical property of experiments*: the probability of getting heads is 0.5 because in a fair coin, heads and tails are equally likely, and this is confirmed by observing that frequency stabilizes near 0.5 in repeated tossing. The textbook's language makes this clear: probability is introduced *after* experiments are performed and frequencies are computed. Probability is not an abstract mathematical object but a numerical description of experimental regularity.

At the senior high level, probability is a *formal measure on a set-theoretic structure*. The axioms define probability as a function P from events (subsets of Ω) to real numbers, satisfying certain conditions. The sample space Ω need not correspond to any physical experiment; events need not be "outcomes" of anything observable. The probability function is defined by its mathematical properties (non-negativity, normalization, additivity), not by its relationship to experimental frequency.

This shift maps directly onto Sfard's (1991) framework. The junior high student has achieved *interiorization*: they can perform experiments, count outcomes, compute frequencies, and apply the classical formula. Some students have reached *condensation*: they can think about the probability of an event without performing the experiment, reasoning about what would happen in principle. But the senior high curriculum demands *reification*: probability must become a structural object, a measure satisfying axioms, defined on an abstract set-theoretic structure.

The difficulty of this reification can be analyzed more precisely. At the junior high level, probability is always *about something*: it describes the likelihood of a physical outcome. The student asks "What is the probability of getting heads?" and answers by thinking about coins, experiments, and frequencies. At the senior high level, probability is a mathematical object in its own right: it can be computed from axioms, compared across different probability spaces, and manipulated using formal rules. The student must treat probability not as a description of experiments but as an entity defined by its mathematical structure.

This ontological rupture generates what Bachelard would call a *substantialist obstacle*. The student, accustomed to thinking of probability as a property of physical experiments, encounters the demand to treat it as an abstract measure satisfying axioms. The junior high conception, in which probability is grounded in experimental frequency, becomes an impediment to the senior high conception, in which probability is defined by mathematical properties independent of any particular experiment.

Consider a concrete illustration. At the junior high level, the question "What is the probability of rolling a sum of 7 with two dice?" is answered by listing all 36 equally likely outcomes, counting the 6 favorable ones, and computing 6/36 = 1/6. The student's reasoning is grounded in the physical experiment: there are two dice, they can land in 36 ways, and 6 of these ways give a sum of 7. At the senior high level, the same question requires defining the sample space Ω as the set of all ordered pairs (i, j) with $i, j \in \{1, 2, 3, 4, 5, 6\}$, defining the event $A = \{(i, j) : i + j = 7\}$, and computing $P(A) = |A| / |\Omega| = 6/36 = 1/6$. The computation is the same, but the conceptual framework is fundamentally different: the student must now think in terms of sets, subsets, and a probability function defined on them.

4.2 The Semiotic Rupture: From Experimental Language to Set-Theoretic Formalism

The ontological shift is accompanied by, and in some sense constituted by, a semiotic rupture. Duval's (1995, 2006) framework allows us to describe this rupture with precision.

At the junior high level, the probability concept is expressed through several semiotic registers:

1. **Natural language (experimental descriptions)**: Tasks ask students to toss a fair coin a specified number of times and record the frequency of heads, or to draw balls from a bag with known contents.
2. **Numerical computation**: $P(A) = m/n = 6/36 = 1/6$.

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3. **Tabular/list representations**: Lists of all possible outcomes; tree diagrams showing branches for each stage of an experiment.

4. **Graphical representations (frequency histograms)**: Bar charts showing the frequency of outcomes across multiple trials.

Throughout the junior high curriculum, the experimental register plays a dominant role. The textbook introduces probability through verbal descriptions of physical experiments, and the definition itself uses natural language grounded in experimental experience, conveying the idea that when an experiment is performed many times, the relative frequency of an event tends to stabilize around a fixed value that serves as its probability. The computations are always linked to specific experiments, and the tree diagrams always represent physical processes (first toss, second toss, etc.).

At the senior high level, new registers appear and quickly become dominant:

1. **Set-theoretic notation**: $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $A \subseteq \Omega$, $A \cup B$, $A \cap B$, $\bar{A}$ (complement).

2. **Random variable notation**: $X: \Omega \to \mathbb{R}$, $P(X = x_i) = p_i$, $E(X) = \Sigma x_i p_i$.

3. **Distribution tables**: Tables listing values of a random variable and their probabilities.

4. **Distribution graphs**: Probability mass functions and cumulative distribution functions plotted on coordinate axes.

The discontinuity between these two semiotic regimes can be analyzed using Duval's concept of *congruence*. At the junior high level, the congruence between registers is relatively high. The verbal description "draw a ball from a bag" corresponds directly to the physical experiment, the list of outcomes corresponds directly to the branches of a tree diagram, and the frequency histogram corresponds directly to the results of repeated trials. The elements of each register map transparently onto the physical experiment that grounds them all.

At the senior high level, the congruence drops sharply. The set-theoretic notation Ω, $A \subseteq \Omega$, $P(A)$ has no direct counterpart in experimental language. The expression "sample space" is not simply a list of outcomes; it is an abstract set whose elements (sample points) may have no physical interpretation. The random variable $X: \Omega \to \mathbb{R}$ is not simply "the number we get from the experiment"; it is a *function* defined on the sample space, mapping each abstract outcome to a real number.

This non-congruence has a concrete consequence. At the junior high level, conversions between registers are *transparent*: the student can see that the verbal description, the tree diagram, the list of outcomes, and the frequency histogram all refer to the same experiment because they are all grounded in the physical situation. At the senior high level, conversions become *opaque*: the student must recognize that $\Omega = \{1, 2, 3, 4, 5, 6\}$ with $P(\{k\}) = 1/6$ for each k, the random variable $X(\omega) = \omega$, and the probability distribution $P(X = k) = 1/6$ all represent the same probabilistic situation, but the common referent is no longer a physical experiment; it is an abstract probability space.

4.3 The Cognitive Rupture: The Frequency Intuition as Epistemological Obstacle

The ontological and semiotic ruptures described above converge at the cognitive level. The frequency intuition conception, which was a genuine cognitive achievement at the junior high level, becomes an *epistemological obstacle* at the senior high level. In the terms of Tall and Vinner (1981), the student's *concept image* of probability, the cognitive structure built from experiments, frequency observations, and concrete events, conflicts with the *concept definition* presented at the senior high level, the axiomatic, set-theoretic formalism. This conflict between concept image and concept definition is a well-documented source of difficulty in advanced mathematical thinking (Tall & Vinner, 1981), and it manifests with particular intensity in the probability domain.

The most fundamental mechanism is what might be called the *empirical anchor*. At the junior high level, every probabilistic question is grounded in a concrete physical situation: a coin, a die, a bag of colored balls. The student asks "What is the probability of getting heads?" and answers by thinking about the

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fairness of the coin and the symmetry of the experiment. This empirical anchor becomes so deeply embedded that the student finds it difficult to think about probability without a physical referent. When the senior high curriculum asks the student to define probability through axioms, to say that P is a function satisfying non-negativity, normalization, and additivity, the axioms are purely formal; they tell you what probability *does*, not what it *is*. For a student whose entire understanding is built on experiments and frequencies, this is genuinely disorienting.

Closely related is the *frequency identification*. The junior high curriculum establishes the idea that frequency stabilizes around the probability through repeated experimentation (People's Education Press, 2024). Over three years of instruction, this identification becomes so natural that the student comes to believe that probability *is* long-run frequency, or at least that the only legitimate way to determine a probability is through repeated experimentation. When the senior high curriculum presents the axioms of probability as the foundation, the student struggles to see how these abstract conditions relate to anything meaningful. The frequency identification functions here as what Bachelard (1938/2002) called an obstacle of first experience: a generalization from a particular kind of encounter (experimental frequency) to a universal conception.

The obstacles extend beyond probability itself. The notion of *event* undergoes a parallel transformation. At the junior high level, an event is something that happens: "getting heads," "rolling a 6." The word carries its ordinary-language meaning: an occurrence, something that takes place in time. At the senior high level, an event is a *subset* of the sample space, a mathematical object that exists timelessly in a set-theoretic universe. The student who thinks of events as happenings will naturally resist the idea that an event can be the empty set ($\emptyset$) or the entire sample space (Ω), since neither corresponds to anything that could "happen" in the ordinary sense.

The concept of *random variable* generates perhaps the most subtle obstacle. At the junior high level, students encounter numerical quantities associated with experiments: the sum of two dice, the number of heads in ten tosses. These quantities vary from one trial to another, and the student understands them as "numbers that come out different each time." At the senior high level, a random variable is not a varying number but a *function*: $X: \Omega \to \mathbb{R}$. This is a categorical shift. The function X does not vary; it is a fixed rule. What varies is the sample point ω, and consequently the value $X(\omega)$. The student must understand that randomness does not reside in the rule but in the input. The term "random variable" is itself misleading: it suggests something that varies randomly, which is precisely what a random variable is *not*.

Finally, there is a shift from *computation* to *distribution*. At the junior high level, the student computes a single probability: $P(A) = m/n$. At the senior high level, the student must work with entire *distributions*, the complete set of values a random variable can take and their associated probabilities. The expected value $E(X) = \Sigma x_i p_i$ and variance $D(X) = \Sigma(x_i - E(X))^2 p_i$ require the student to consider the random variable as a whole, weighing all its possible values simultaneously. This shift from a single value to an entire distribution is a manifestation of Sfard's (1991) reification: the random variable must become an object of thought rather than a process of obtaining a number.

These obstacles are not independent. The empirical anchor reinforces the frequency identification; the frequency identification makes the axiomatic approach seem arbitrary; the concrete event conception makes the set-theoretic framework seem unnatural; the number conception of random variable blocks the functional understanding. Together, they form a web of interlocking obstacles that make the transition one of the most difficult concept transitions in the secondary school curriculum.

4.4 The Rupture as Structurally Necessary: Historical and Logical Arguments

The analysis above reveals that the transition from frequency intuition to random variable formalism is not a smooth deepening of understanding but a genuine rupture. This raises a question: is this rupture a contingent feature of the Chinese curriculum, or does

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it reflect a deeper structural necessity inherent in the probability concept itself?

We argue for the latter, while acknowledging that the particular form and intensity of the rupture is shaped by curricular choices. The probability concept, as it has developed in the history of mathematics, necessarily involves discontinuities. Each major stage in the concept's historical evolution was overcome through a rupture, not because the preceding conception was "wrong" but because its internal limitations became untenable. Examining these historical ruptures in detail illuminates why the curriculum transition demands a comparable break.

4.4.1 The first rupture: From classical probability to frequentism

The classical conception, crystallized in Laplace's (1814) *Essai philosophique sur les probabilités*, defined probability as the ratio of favorable cases to all equally possible cases. This definition is intuitive and computationally effective for symmetric situations like dice and cards, but it harbors a fatal circularity: it defines probability in terms of *equally possible* outcomes, where "equally possible" itself presupposes a notion of probability (equal probability). The definition works when our intuitions about symmetry are clear: a fair coin has two "equally possible" outcomes, but it offers no principled criterion for determining when outcomes are equally possible in ambiguous situations.

This circularity was not merely a logical curiosity; it generated concrete paradoxes. Bertrand's (1889) paradox demonstrated that the classical definition yields contradictory answers to the same question depending on how "equally possible" is parameterized: the probability that a random chord of a circle is longer than the side of an inscribed triangle takes three different values (1/3, 1/2, 1/4) depending on whether one considers the chord's endpoints, its midpoint, or its distance from the center as the "equally possible" variable. The classical conception had no internal resources to resolve this contradiction because it could not distinguish between different parameterizations of the same sample space.

The frequentist conception, probability as the limit of relative frequency in a long sequence of identical trials, as formalized by von Mises (1928/1957), resolved this circularity by grounding probability in observable, repeatable experiments. Under frequentism, probability is not a ratio of possibilities but an empirical limit: toss a coin enough times and the proportion of heads will converge to a definite number. This was a genuine rupture with the classical conception: probability was no longer a property of the *setup* (equally possible outcomes) but a property of the *long-run behavior* of a repeatable process. The student's frequency intuition conception, developed in junior high school, is structurally analogous to this frequentist stage.

4.4.2 The second rupture: From frequentism to axiomatization

The frequentist conception was itself overcome by internal pressures. Its most conspicuous limitation was the inability to assign probabilities to non-repeatable events: What is the probability that a specific earthquake will occur in the next decade? What is the probability that a particular scientific hypothesis is true? These questions are meaningful and practically urgent, but frequentism cannot address them because there is no sequence of identical trials to which they refer. Beyond this practical limitation, frequentism faced deep theoretical difficulties. The definition of probability as a *limit* of relative frequencies required specifying what kind of limit, under what conditions, and with what guarantees of convergence. The mathematical apparatus needed to make these notions precise (measure theory, developed by Lebesgue, Borel, and others in the early twentieth century) revealed that the intuitive notion of "frequency" was far more mathematically complex than the frequentists had supposed.

Kolmogorov's (1933) *Grundbegriffe der Wahrscheinlichkeitsrechnung* resolved these difficulties through a second rupture: probability was detached from any particular interpretation and defined purely as a measure satisfying three axioms (non-negativity, normalization, and countable additivity) on a measurable space. Under this conception, probability is neither a ratio of possibilities nor a limit of frequencies; it is a mathematical function whose properties are determined by its axioms. The connection to empirical reality is no longer built into the definition but must be established through interpretation (frequentist, Bayesian, propensity, or other). This was not merely a generalization of frequentism; it was a transformation of what probability *is* as a mathematical object.

The senior high school curriculum's introduction of sample spaces, events as sets, probability as a measure on those sets, and random variables as functions on the sample space recapitulates this second historical rupture. The student who has internalized the frequentist conception, that probability is what frequency stabilizes to in repeated experiments, must now accept that probability is a formal measure whose existence and properties are guaranteed by axioms rather than by empirical convergence.

4.4.3 The parallel with student cognition

The student's cognitive development traverses a sequence structurally analogous to these historical ruptures. The junior high frequency intuition conception is the student's "classical" stage: probability is computed by counting equally likely outcomes in concrete experiments. The more sophisticated experimental frequency conception, that probability is what frequency approaches in the long run, is the "frequentist" stage. The senior high axiomatic conception, with sample spaces, set-theoretic events, and random variables, is the "Kolmogorov" stage.

This parallel should not be read as a naïve recapitulation thesis. Individual learning does not simply replay the history of the discipline. Rather, the parallel exists because the logical structure of the probability concept creates similar demands at each level. At each stage, the learner (or the mathematical community) has achieved something real and valuable: a working conception that supports a range of problems. At each stage, that conception encounters situations it cannot handle. And at each stage, the resolution requires not extending the old conception but replacing it with a fundamentally different one.

This parallel also illuminates why the rupture is not merely a matter of adding sophistication. The classical conception was not a less precise version of the frequentist conception; it was a *different kind of thing* (counting possibilities vs. observing long-run behavior). The frequentist conception was not a less precise version of the axiomatic conception; it was a *different kind of thing* (empirical limit vs. formal measure). Similarly, the frequency intuition is not a rough approximation of the random variable conception; it is a fundamentally different way of understanding what probability is.

4.4.4 Implications of the structural argument

If the rupture is structurally necessary, several consequences follow for curriculum design and pedagogy. First, the rupture cannot be eliminated by better sequencing or smoother transitions; it can only be made more navigable. Second, the curriculum should acknowledge the discontinuity explicitly rather than pretending that the senior high framework is simply a "more rigorous version" of the junior high approach. Third, the historical narrative, the actual sequence of crises, paradoxes, and reconstructions through which the probability concept developed, can serve as a pedagogical resource. By situating the student's own transition within this history, educators can help students understand the rupture not as a failure of their junior high learning but as a natural step in the development of mathematical thought.

This structural account does not mean that all students must experience the rupture in the same way. The particular form of the rupture is shaped by the curriculum, by teaching, and by the student's own cognitive resources. A curriculum that explicitly acknowledges the discontinuity and provides transitional support may mitigate its intensity. But the rupture itself, the need for a fundamental restructuring of what probability is understood to be, is not an artifact of poor curriculum design. It is an inherent feature of the probability concept, rooted in the same intellectual tensions that drove the historical development of the discipline.

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V. DISCUSSION

5.1 Implications for Teaching

If the transition from frequency intuition to random variable formalism is an epistemological rupture rather than a smooth progression, then pedagogical approaches must address the rupture at its source. Simply presenting the new definition and providing more practice will not resolve a conflict between concept image and concept definition (Tall & Vinner, 1981). What is needed is a deliberate strategy of what Bachelard called *psychoanalysis of objective knowledge*: making the prior conception explicit, examining where it works and where it breaks down, and thereby creating the conditions for its restructuring.

At the senior high level, this might begin with problems that deliberately expose the limitations of the frequency conception. A teacher could pose questions such as the probability of a specific earthquake occurring in the next decade or a new drug being approved by regulators. Such questions involve events that cannot be repeated under identical conditions, so the frequency-based definition offers no guidance. The point is not to provide answers but to create a cognitive need for a more general framework, a need that the axiomatic approach can then satisfy.

The random variable concept deserves particular attention, given the subtlety of the shift from "a number that varies from trial to trial" to "a function defined on the sample space." A productive pedagogical sequence might proceed through several stages. First, students define informal rules that assign numbers to experimental outcomes ("Let X be the number of heads"). Then, the teacher makes the functional character of the rule explicit by listing several outcomes and their corresponding values, creating a table that shows X as a mapping rather than a varying quantity. Only at this point is the formal notation $X: \Omega \to \mathbb{R}$ introduced, as a way of capturing something the student has already been doing informally. The key insight is that X itself is not random; it is a fixed rule; what is random is which sample point ω is realized, and therefore what value $X(\omega)$ takes.

Duval's (1995, 2006) theory further suggests that conversion between semiotic registers should be a deliberate focus of classroom instruction. Students will need to move between verbal descriptions of experiments, set-theoretic notation, random variable notation, distribution tables, and distribution graphs. Because the congruence between these registers is low (as shown in Section 4.2), such conversions cannot be left to implicit acquisition. Teachers should design tasks that require explicit conversion: given a verbal description, write the set-theoretic formulation; given a distribution table, draw the probability histogram; given a random variable definition, construct the distribution table.

5.2 Implications for Curriculum Design

While teaching strategies can mitigate the rupture, the structural features of the curriculum themselves contribute to its severity. The Chinese curriculum divides probability teaching sharply between the junior high and senior high levels, with distinct textbooks, distinct standards documents, and (in practice) distinct teachers. This institutional separation reinforces the impression that the two levels represent fundamentally different enterprises rather than stages in a continuous, if discontinuous, development.

Curriculum designers could introduce elements of continuity at the points of greatest discontinuity. At the junior high level, the language of "sample space" could be introduced informally, using set-bracket notation to list outcomes, before the full set-theoretic framework is required. The 2024 PEP-JH textbook already lists outcomes systematically but does not explicitly connect this practice to the set-theoretic language that students will later encounter. Introducing Ω as a label for the set of all outcomes at the junior high level would create a semiotic bridge without requiring a conceptual rupture.

At the senior high level, the curriculum could present the axiomatic framework not as a *replacement* for the junior high conception but as a *generalization* that explains why the junior high approach worked in the cases where it did. The historical development of probability, from Laplace's classical conception through von Mises' frequentism to Kolmogorov's axiomatics, provides a natural narrative for this progression. By situating the student's own transition within this historical sequence, the curriculum can help students understand the rupture not as a failure of their junior high learning but as a natural step in the development of mathematical thought.

The treatment of random variables in the current curriculum also warrants structural revision. The PEP-SH textbook moves rapidly from the formal definition $X: \Omega \to \mathbb{R}$ to probability distributions and expected values. A more gradual progression through intermediate stages would better support this transition: (1) informal numerical summaries of experiments, (2) explicit rules that map outcomes to numbers, (3) formal definition as functions, and (4) distributions and distributional properties. Such a sequence would align more closely with Sfard's (1991) stages of interiorization, condensation, and reification, providing the cognitive support that each stage requires.

5.3 Implications for Research

This paper has offered a theoretical analysis of the epistemological rupture in the probability concept transition. The analysis generates several questions for future empirical research.

First, how do students actually work through the transition from frequency intuition to random variable formalism? Do they exhibit the patterns of obstacle and rupture predicted by the theoretical framework? Classroom-based studies using think-aloud protocols, clinical interviews, and task-based analysis could reveal the specific ways in which prior knowledge serves as an obstacle. The representativeness heuristic, for example, has been shown to influence junior high students' probability reasoning in systematic ways (He, 2020), and it would be important to investigate whether such heuristics persist and interact with the formal conceptions introduced at the senior high level.

Second, what specific instructional interventions are most effective in supporting students through the rupture? Can task sequences designed with the three dimensions of the rupture in mind improve student outcomes? Recent methodological advances in tracing learning through combined process-object theories and individual learning trajectories (Bednorz et al., 2026) offer promising tools for such investigations.

Third, how does the rupture manifest in different curriculum systems? The Chinese curriculum provides a particularly sharp case, but analogous transitions exist in other systems. Comparative studies could illuminate which features of the rupture are universal and which are curriculum-specific.

Fourth, how do teachers' own conceptions of probability influence their treatment of the transition? Research on teachers' mathematical knowledge for teaching probability (Batanero et al., 2016) suggests that teachers' own understanding of the relationship between frequency and probability may affect how they mediate the transition.

5.4 Limitations

This study is a theoretical analysis, not an empirical investigation. The claims about student cognition are based on theoretical reasoning and on the analysis of textbooks and curriculum standards, not on direct observation of student learning. The identification of specific obstacles is derived from the theoretical frameworks (Bachelard, Duval, Sfard) rather than from systematic data collection. Future empirical research is needed to test and refine the theoretical claims made here.

Additionally, this analysis focuses on the probability concept as presented in two specific textbook series (PEP-JH 2024 and PEP-SH 2019). Other textbook series used in China, such as the Beijing Normal University Press (北师大版) edition, may present the transition differently. The role of teachers as mediators of the curriculum is also not addressed, although it is likely to be significant in practice.

The theoretical frameworks employed each carry their own assumptions and limitations. Bachelard's philosophy of science, developed primarily through analysis of physics and chemistry, may not transfer transparently to mathematics education. Duval's theory has been criticized for its limited

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treatment of social and contextual factors. Sfard's claim that reification is a sudden rather than gradual process has been questioned. The convergence of these three frameworks, while productive, does not guarantee that the resulting analysis captures all relevant dimensions of the transition.

Finally, the historical parallel drawn in Section 4.4 should be read as an illuminating analogy rather than a strict recapitulation thesis. The relationship between individual learning and historical development is contested in mathematics education research.

VI. CONCLUSION

The concept of probability occupies a peculiar position in school mathematics. It is at once the most accessible of mathematical ideas, as everyone has an intuitive sense of what it means for something to be "likely," and the most resistant to formal treatment. The transition from junior high school to senior high school in the Chinese mathematics curriculum brings this tension to a head. Students who have spent three years understanding probability through experiments, frequency, and counting must now reconstruct their understanding in terms of sample spaces, events as sets, and random variables as functions.

This paper has argued that the difficulty of this transition is not incidental but structural. At the ontological level, probability must shift from being a property of experiments to being a measure on an abstract structure. At the semiotic level, the student must master a new system of notation (set-theoretic, functional, distributional) that has low congruence with the registers of the junior high curriculum. At the cognitive level, the frequency intuition conception that served the student well becomes an obstacle to the new formal conception, operating through multiple interlocking mechanisms: the empirical anchor, the frequency identification, the concrete event conception, the number conception of random variable, and the computational conception of distribution.

These dimensions are not additive; they interact. The semiotic rupture amplifies the cognitive obstacle (the unfamiliar notation makes the new conception harder to grasp, which makes the student retreat to the familiar frequency intuition). The cognitive obstacle intensifies the ontological rupture (the persistent concept image of probability-as-frequency makes it difficult to see what the axioms are for). Understanding these interactions is essential for designing instruction that addresses the rupture as a whole rather than its parts in isolation.

The practical implication is that the transition cannot be managed by presenting the new framework and expecting students to adapt. It requires a deliberate pedagogical strategy: problematizing the frequency conception before replacing it, building semiotic bridges between the old and new registers, and supporting the reification of probability from a process (observing frequencies) into an object (a measure satisfying axioms). The curriculum, too, has a role to play: not in eliminating the rupture (which is structurally necessary) but in making it more navigable.

Bachelard observed that the human mind has a tendency to accept the familiar as natural, resisting the restructuring that new knowledge demands, and thus needs to be surprised into intellectual progress (Bachelard, 1938/2002). The frequency intuition is, in this sense, a kind of intellectual comfort zone. The student who has learned to compute probabilities through experiments and counting has achieved something real and valuable. The difficulty begins when this achievement becomes a cage, a way of thinking that prevents the student from seeing what probability could become. Recognizing this rupture not as a defect but as a necessary feature of mathematical development is the starting point for more coherent instruction at one of the most critical junctures in the school mathematics curriculum.

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